Q: $a^i b^{j^2} \mid i,j \geq 1$

$a^i = \{a, aa, aaa \ldots\}$ — FA ✓

$b^{j^2} = \{b, b^4, b^9 \ldots\}$ — FA ✗

RL ✗

Q: $a^i b^{2^n}$ RL ✗

a^i → FA ✓, b^{2^n} → FA ✗

Eg: $a^i b^p \mid i \geq 1$, p is prime

RL ✗

Eg: $w \mid n_a(w) = n_b(w)$ $\Sigma = \{a, b\}$

a's b's

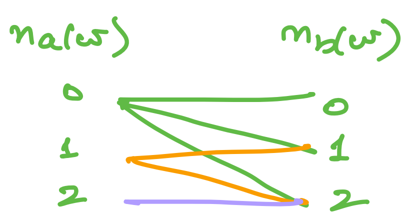
store a count
store b count } check =

FA ✗ RL ✗

Eg: $w \mid n_a(w) \leq n_b(w)$
Eg: $w \mid n_a(w) \geq n_b(w)$ } counting not possible in FA

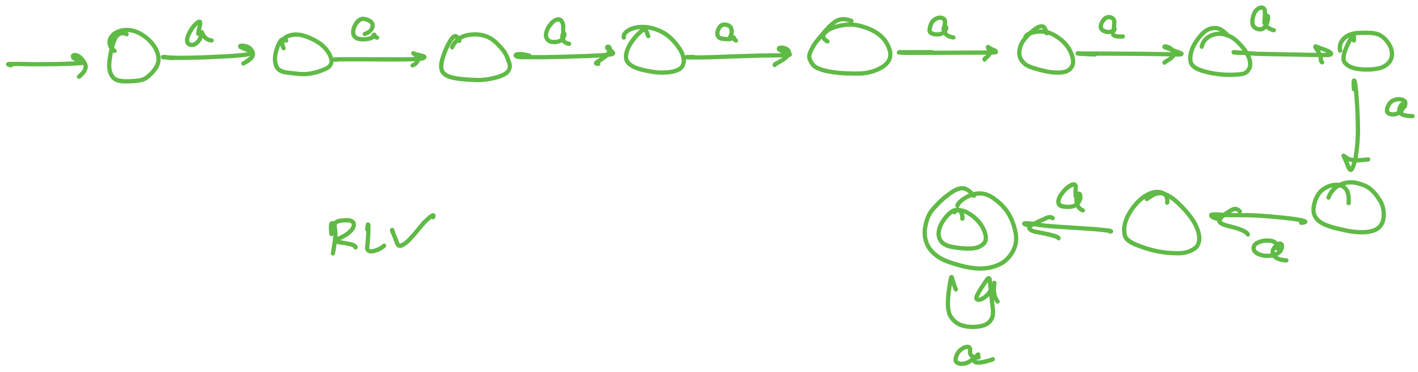
RL ✗

Eg: $n_a(w) \bmod 3 \leq n_b(w) \bmod 3$



FA can do modular counting

Eg: $a^n \mid n \geq 10$ no of a's count atleast 10



RL ✓

Eg: $w w w^R \mid w \in (a,b)^*$

w w w^R

Not Regular

RL ✗

Eg: $w x w^R \mid w, x \in (0,1)^*$

RL ✓

w x w^R

x grow

w x w^R

RE: $0(0+1)^+0 + 1(0+1)^+1$

Eg: $w x w^R \mid w \in (0,1)^*$

$|x| = 5$

↳ you can't extend x beyond length 5

Not Regular

RL ✗

Eg: $a^n b^{n+m} c^m \mid n, m \geq 1$

a's b's b's c's

n n m m

FA ✗

RL ✗

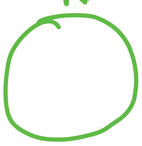
Closure Properties of Regular language

RL are closed under union, intersection, concatenation, complementation & kleene closure

closed:

Set of all natural nos

N



2 natural nos

3 5

3 ∈ N

5 ∈ N

operation: addition +

3 + 5 = 8 8 ∈ N

↳ Result is also a natural no.

Natural nos are closed under addition

if R3 ∈ RL then we will say RL are closed under op operation

Union

L_1 & L_2 are RL

$L_1 \cup L_2$ will also be Regular

If L_1 is Regular then R_1 is RE corresponding to it.

If L_2 is ________ R_2 ________

$L_1 \cup L_2 \rightarrow R_1 + R_2$

$L_1 \rightarrow 0(0+1)^*0 + 1(0+1)^*1 + 0 + 1$

$L_2 \rightarrow 0(0+1)^*1 + 1(0+1)^*0$

$\Big\} \; L_1 \cup L_2$

Concatenation:

$L_1 \quad L_2$ are RL

RE: $\quad L_1 \downarrow R_1 \quad L_2 \downarrow R_2$

$L_1 \cdot L_2 \longrightarrow R_1 \cdot R_2$

Kleene Closure

L_1 is a RL

$\downarrow$

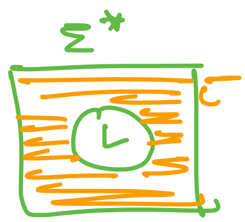
R_1 is a RE

$L_1^* \longrightarrow R_1^*$

R_1^* is also Regular

Complementation:

L is a RL



$\bar{L}$ is also a RL

L is a RL $\longrightarrow$ DFA $\longrightarrow (Q, \Sigma, \delta, q_0, F)$

$\bar{L}$ $\qquad$ DFA $\longrightarrow (Q, \Sigma, \delta, q_0, Q-F)$

$\downarrow$

Create a DFA

Hence, it is a RL

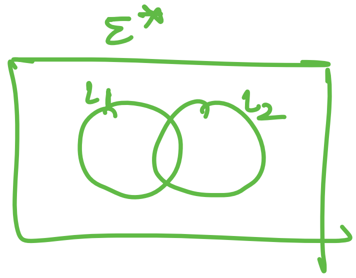
Intersection

$L_1 \cap L_2$

$\downarrow \quad \downarrow$

RL $\quad$ RL

$L_1 \cap L_2 = \overline{\overline{L_1} \cup \overline{L_2}}$



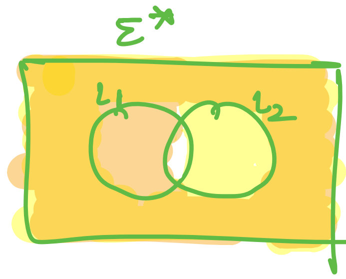
$\overline{L_1}$ is Regular

$\overline{L_2}$ is Regular

$\overline{L_1} \cup \overline{L_2}$ is Regular

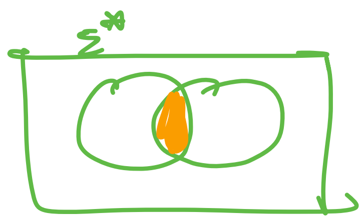
$\overline{\overline{L_1} \cup \overline{L_2}}$ is Regular

$L_1 \cap L_2$ is Regular



$\overline{L_1} \cup \overline{L_2}$

$\overline{\overline{L_1} \cup \overline{L_2}}$



Difference

$L_1 - L_2$

$L_1 - L_2 = L_1 \cap \overline{L_2}$

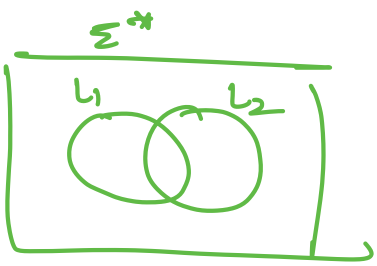
L_1 is a RL

L_2 is a RL

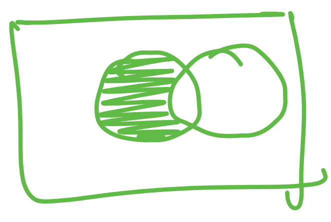
$\overline{L_2}$ is a RL

$L_1 \cap \overline{L_2}$ is a RL

$L_1 - L_2$ is a RL



$L_1 - L_2$



Reversal

$L \rightarrow RL$

$L^R \rightarrow RL$?

$L \rightarrow$ Regular $\rightarrow$ DFA

$\downarrow$

Reverse DFA

$\downarrow$

FA

L^R

DFA $\xrightarrow{\text{Reverse}}$ FA

(L) $\qquad$ (L^R)

Regular languages are not closed under ∞ Union:

$L_1 = \{a^1 b^1\}$

$L_2 = \{a^2 b^2\}$ $\Big\}$ Regular

$L_3 = \{a^3 b^3\}$

$L_1 \cup L_2 \cup L_3 \; \ldots\ldots\ldots = \{a^n b^n \mid n \geq 1\}$

not Regular